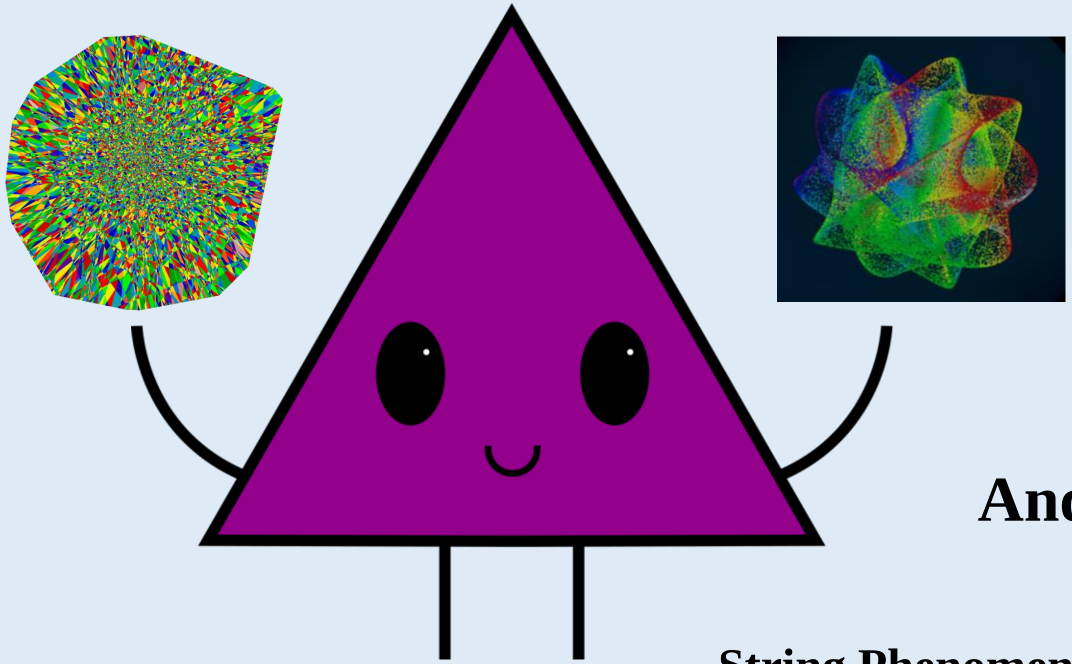
Convergence of Worldsheet Instanton Corrections in AdS Flux Vacua



Andres Rios-Tascon

Cornell University

String Phenomenology conference 2022

Based on work with M. Demirtas, M. Kim, L. McAllister, J. Moritz.

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Takeaways:

- Our improvements in computing Gopakumar-Vafa invariants allow for more complex constructions and robust checks of control (among many other things!).
- These tools will be integrated into our CYTools package, which will be released soon.

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Gukov-Vafa-Witten flux superpotential

Non-perturbative contributions from ED3-branes or strong gauge dynamics on stacks of seven-branes.

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In our flagship example we have

$$W_{\text{flux}}(\tau) \propto -2e^{2\pi i \tau \cdot \frac{7}{29}} + 252 e^{2\pi i \tau \cdot \frac{7}{28}} + \mathcal{O}\left(e^{2\pi i \tau \cdot \frac{43}{116}}\right)$$

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Numbers in red are Gopakumar-Vafa (GV) invariants.

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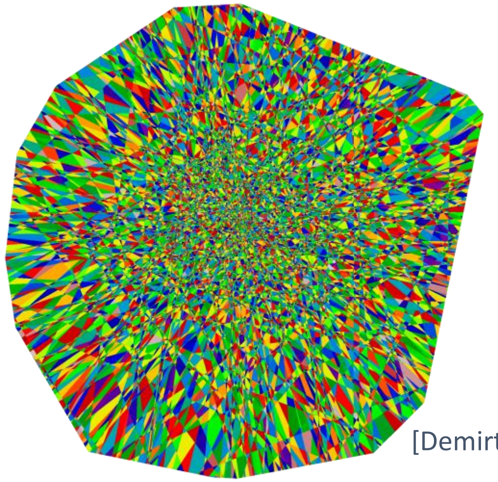

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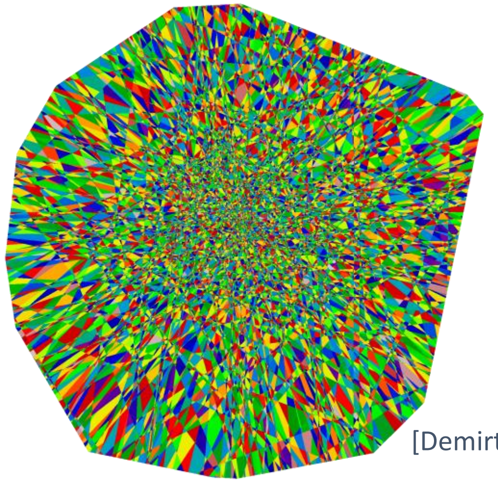

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Et voilà!

We have SUSY AdS with small cosmological constant and all moduli stabilized.

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We must make sure that the vacuum is in the radius of convergence, and that we can find a new point in Kähler moduli space where $\mathrm{D}_{\mathrm{T_i}}\mathrm{W} = 0$ with the corrected volumes.

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   With the standard procedure it is impossibly difficult to look deep into rays, so we needed to come up with some tricks.

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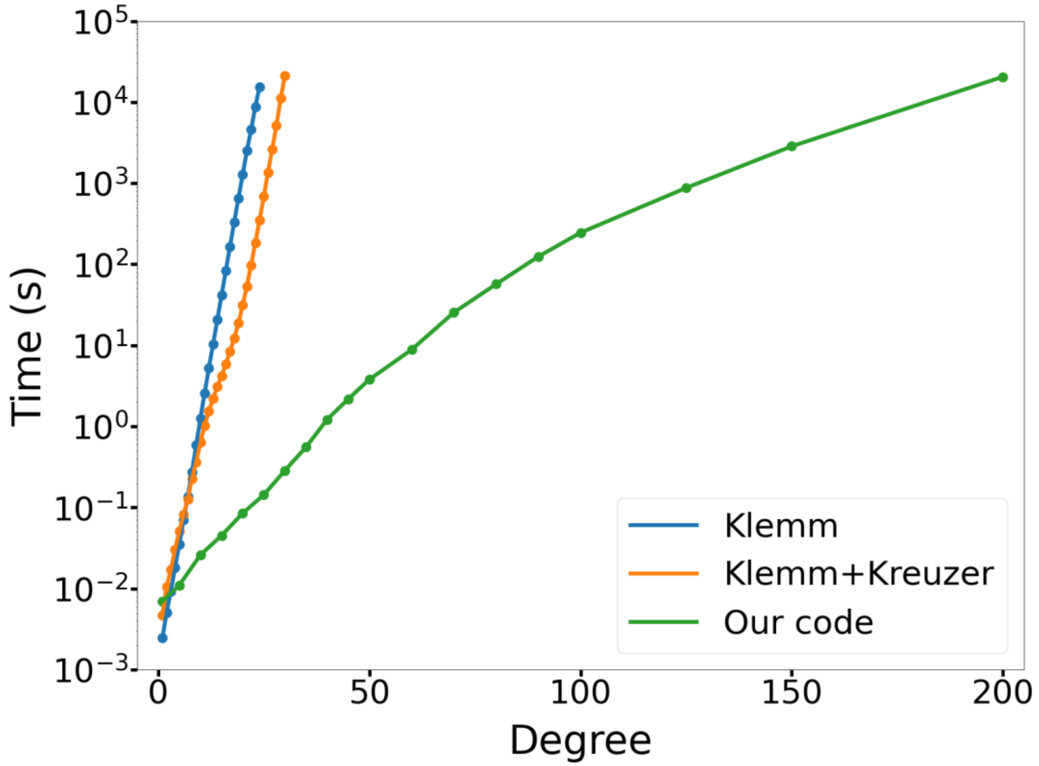

Here is a comparison for an example at $h^{1,1} = 2$, where the Instanton package can be used.

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-2, 286, 1, 1, 4, 1, 188454, 1, 7, -32, -32, 1695, -3038, -4, -2, 5, 1, 1, 1, 1, 392084, 1, 286, 1, 1, -4, 286, -4, -3038, 7, 1, 1, 1, 1, -3038, -84302270, -4, 7, 1, 6073311, 1, -10, -400, 5, 1, 27, -17687468032, 1, 1, -32, 1, 197287568723655, 5187, 5187, 35870, 9, -4, -110, -4, 35, -2927443754647296, -84302270, -4, 1, -72384, 1, 264593385735, 4, 1206291308, 35870, -110, -6196718, -2, 35870, 35, -3038, 1, 35870, -2, 64, -2228160, -72384, 35, -454880, -32, -454880, 1, 1, 1, -6, 1059649, 1, 1, -4, 286, 1206291308, 1, 1, -572, 1, 1651, -3038, 5, -4, 35, 7, 1, 1, -2, -400, -2, -2, -288, -4, 1, 35, 35, -2, 1, 99337500, -454880, 1, -2, -4024945917314, 44000514720961743, -17687468032, 135, -110, 1, -400, -2, 1651, -2, -3038, -17064, 6073311, 6073311, 1, 1, -4, 286, 6073311, 6076, -32, 1059649, 35, -1611792000, 5, 62101640836955, -110, 264593385735, 1, -400, -2, 1, 1, -668908727886779298, 35870, 1, -16043632, -4, 286, 1, 1, 1, -4, -400, 35, -25216, 1, 1, 1, 6885, 5187, -454880, -400, -32, 5, 1, -17687468032, 27748899, 1, 1, 1, 1, -2, -2, 1, 5, -4, 5, 1, -4, -6, 7, 7, -146718, 1, 5187, 1, 3, 5187, 1, 1, 1, 286, 8, 35, 35, 135, -2, 1, -4, -6, -2, -84302270, -4, -84302270, -84302270, -4, -4024945917314, 10272581487272296287, -969921269646560, 264593385735, 26421445359, 35, -2592, 249045000, -16043632, -2, -71740, 1, 1, 35, -3038, 1651, -72384, 35870, 392084, 35, -4, 5187, -2, -400, 1059649, -25216, 47775, 9, -6, 2953818, 35, 135, 15309505269479942, 1206291308, -6196718, 6073311, -72384, -400, -436925483986, 909760, 249045000, 1206291308, 1206291308, -4024945917314, -360012150, -2, 1, -159199764298612184400, 35, -70, -4, 7, 62101640836955, -72384, 1, 1, -454880, -32, -3940930812, -72384, 1, -25216, -110, 1, -4, -2, -3038, 4, -2592, -32, -400, 1, 1, -2, -2, -400, 1, 35, 1, 5187, -4, -2, -192, -36

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1, -2, 1, 1, 1, 5, 1, 1, -32, 286, 1, 5, -32, -6, -2, 1, 35870, 1, -360012150, 1, 1, 1, 1, 1, 1695, 1, 7, 1, 1, -2, 1,
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4, 7, 62101640836955, -72384, 1,

We can go all the way up to $h^{1,1} = 491$, and use 100+ million curve classes.

4, -2592, -32, -400, 1, 1, -2, -2,

Why is this a difficult check?

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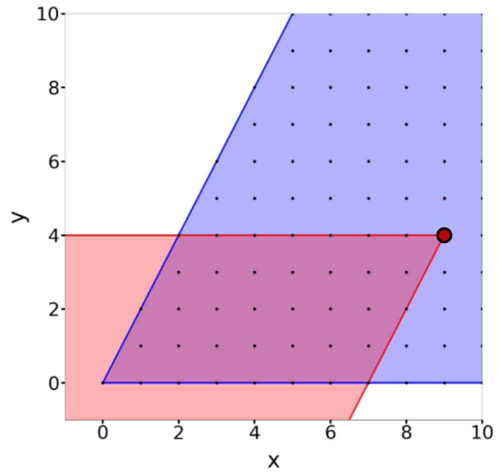

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Blue region is the Mori cone.

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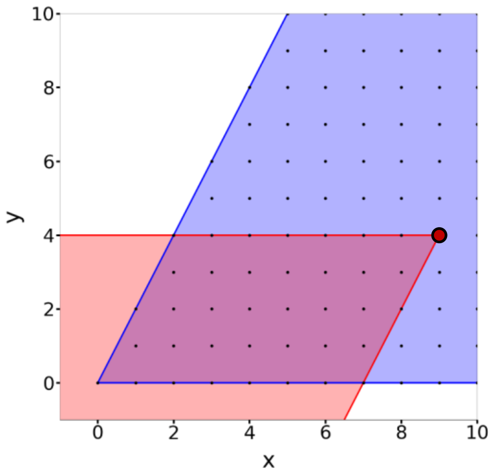

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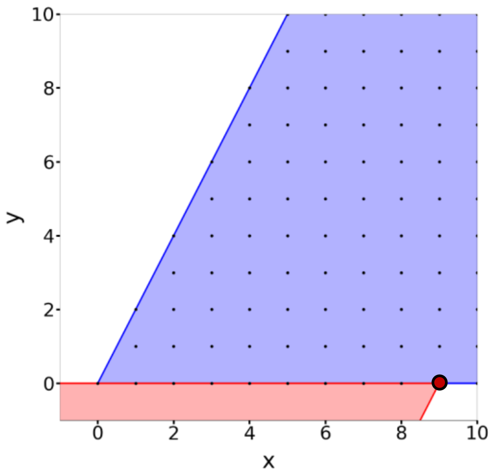

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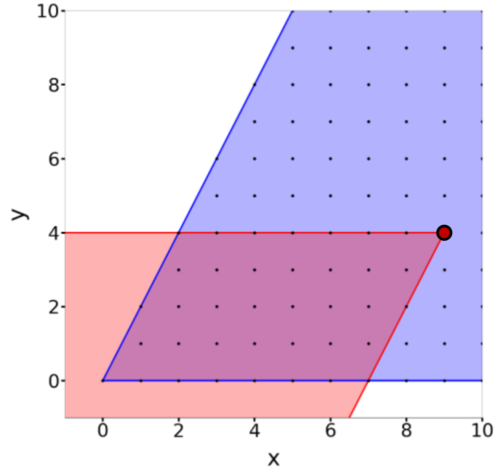

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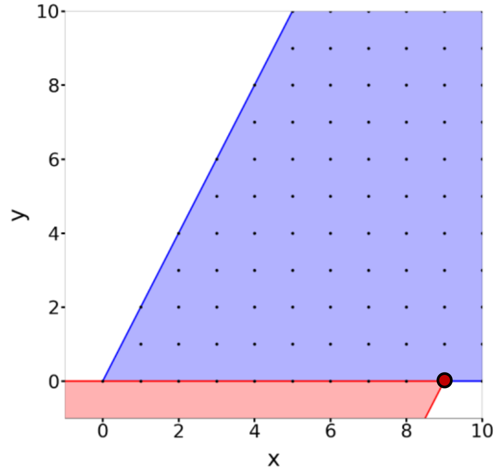

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Computing GV invariants on a d-dimensional face is about as difficult as computing GV invariants for a model with d moduli.

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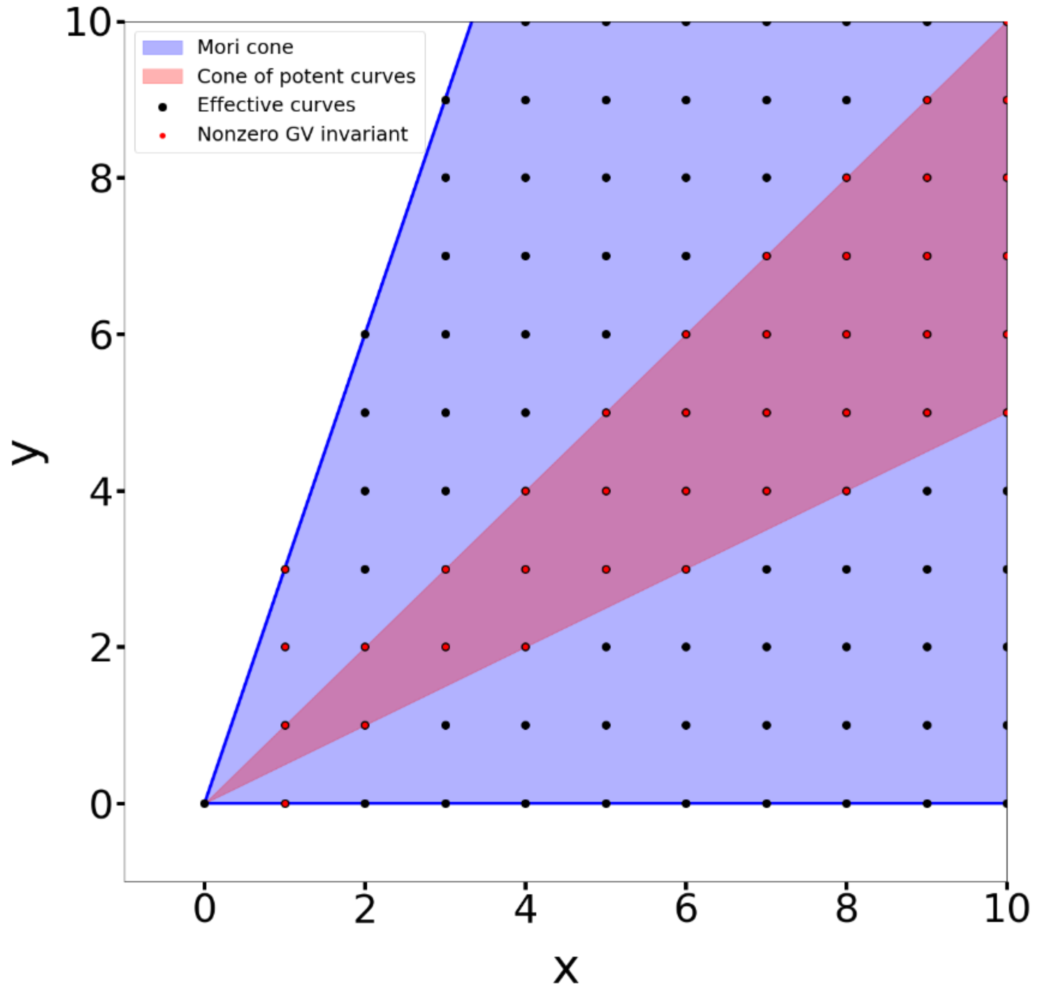

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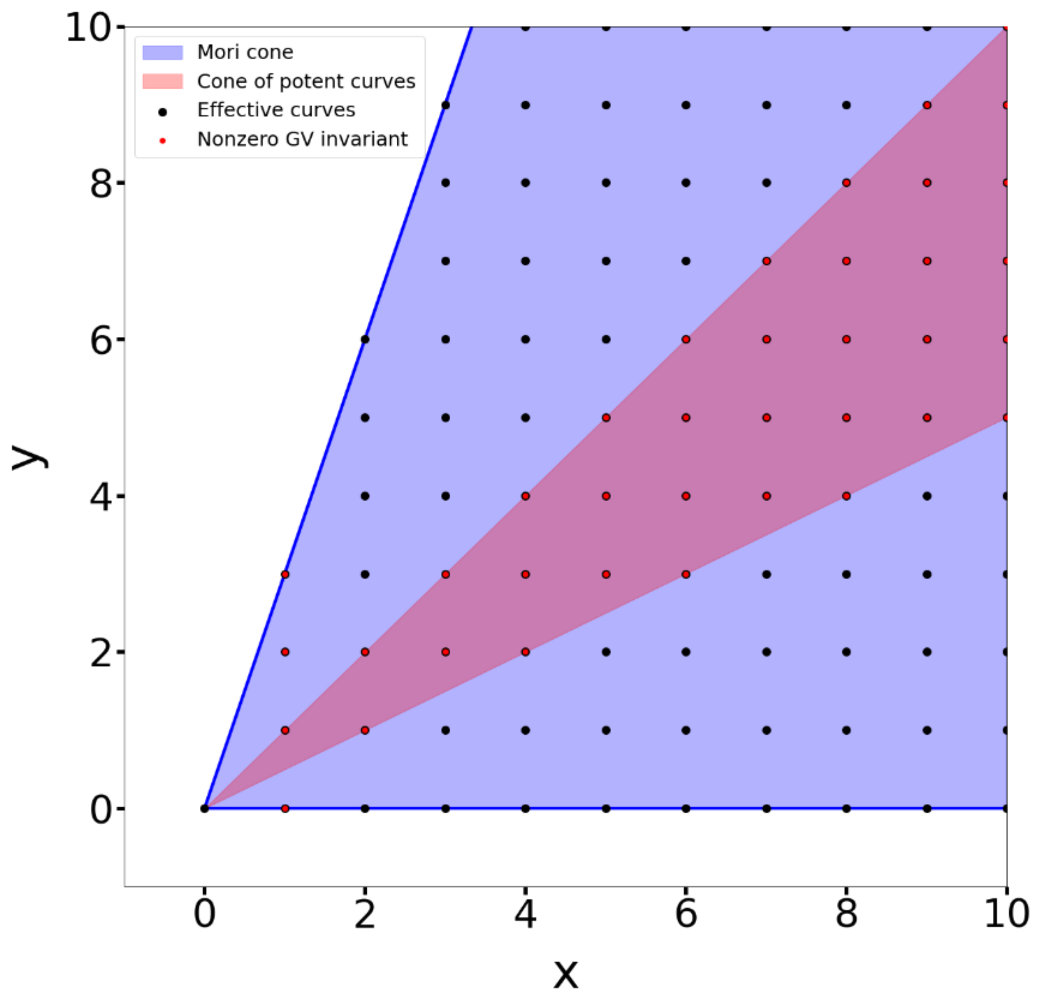

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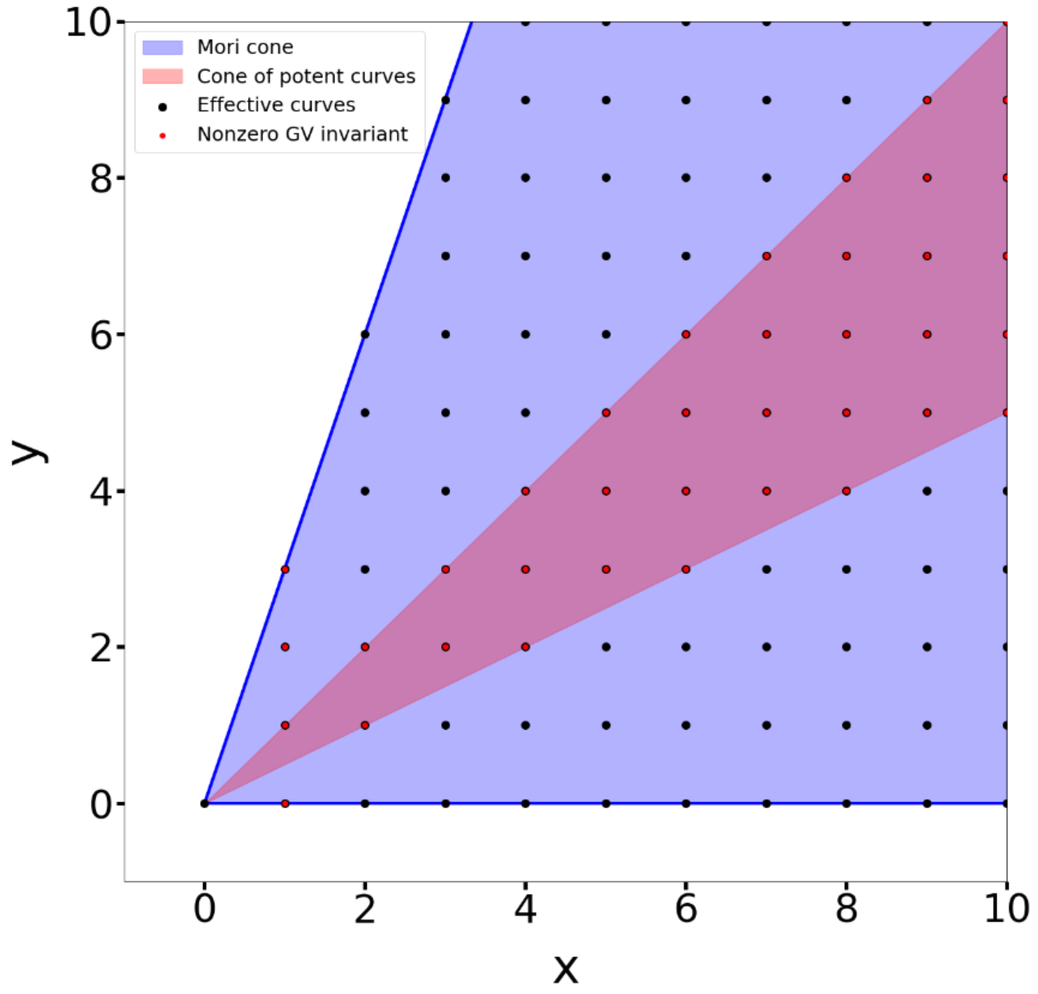

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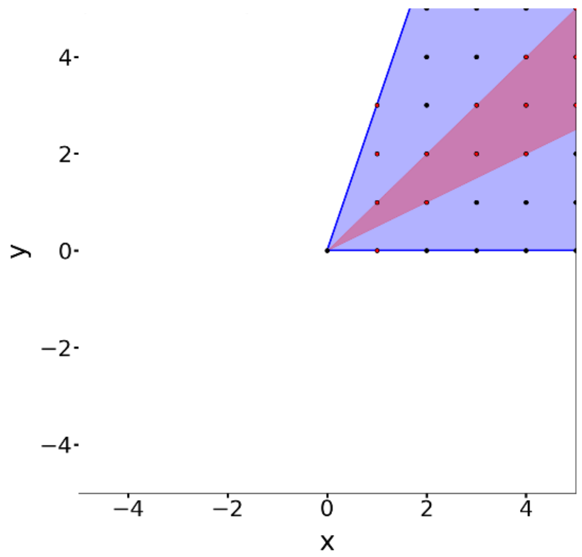

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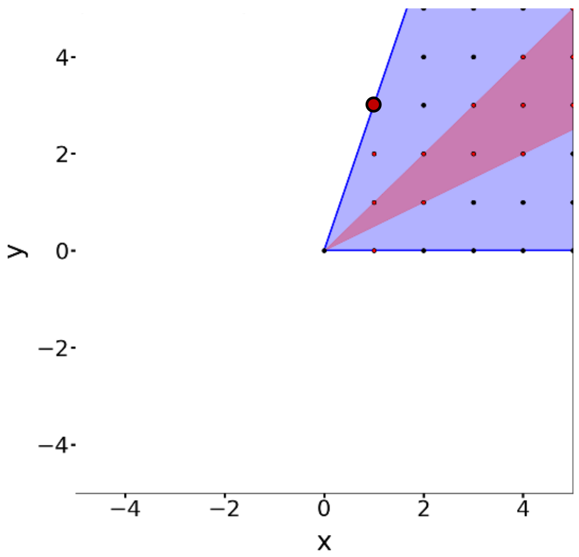

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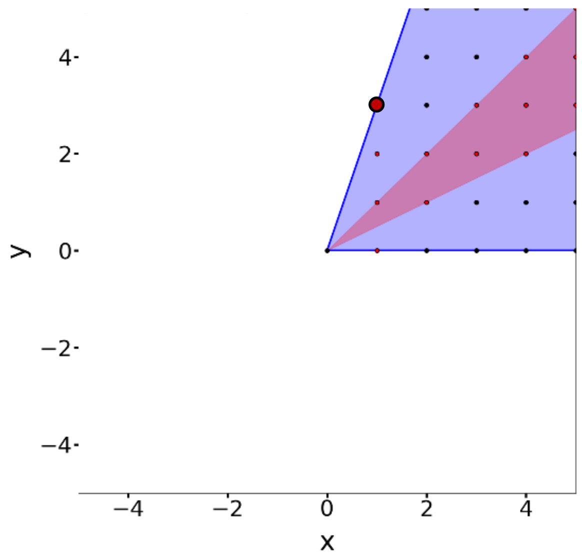

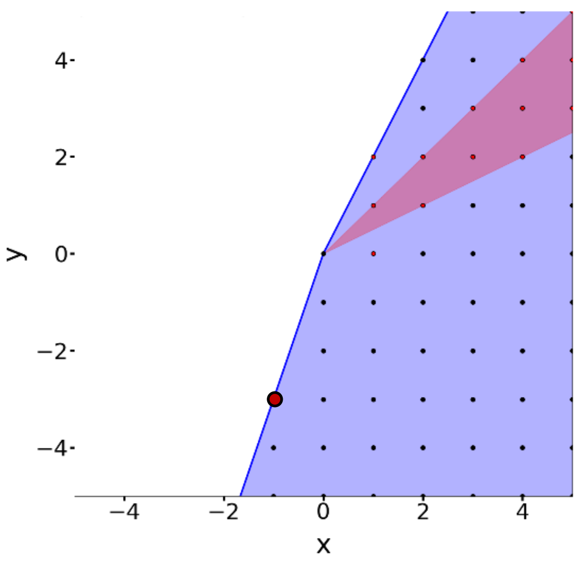

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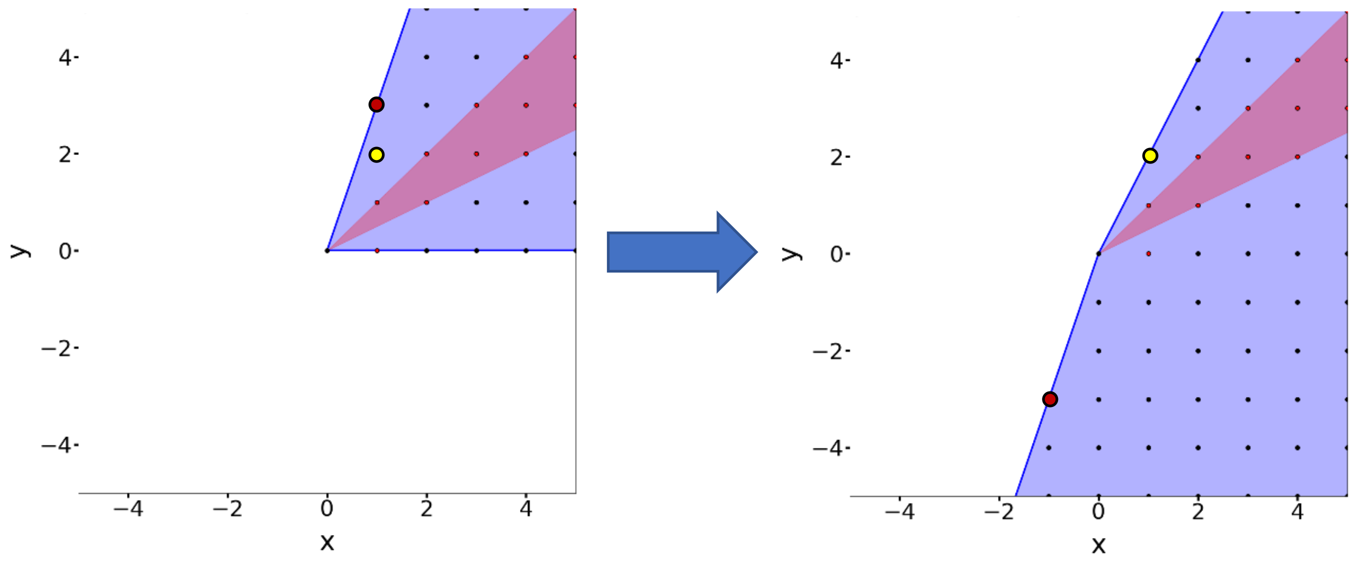

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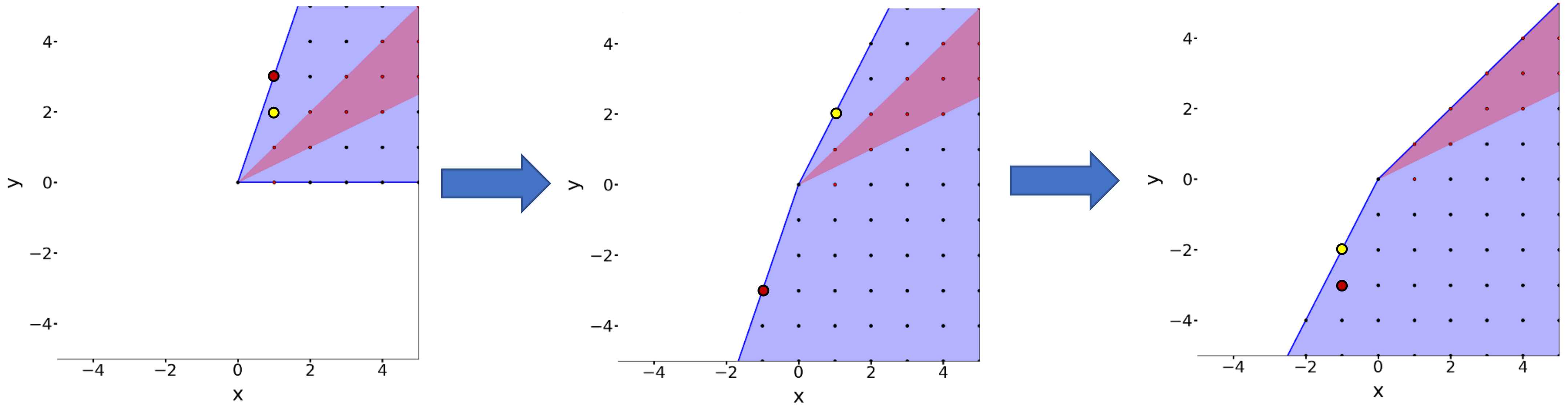

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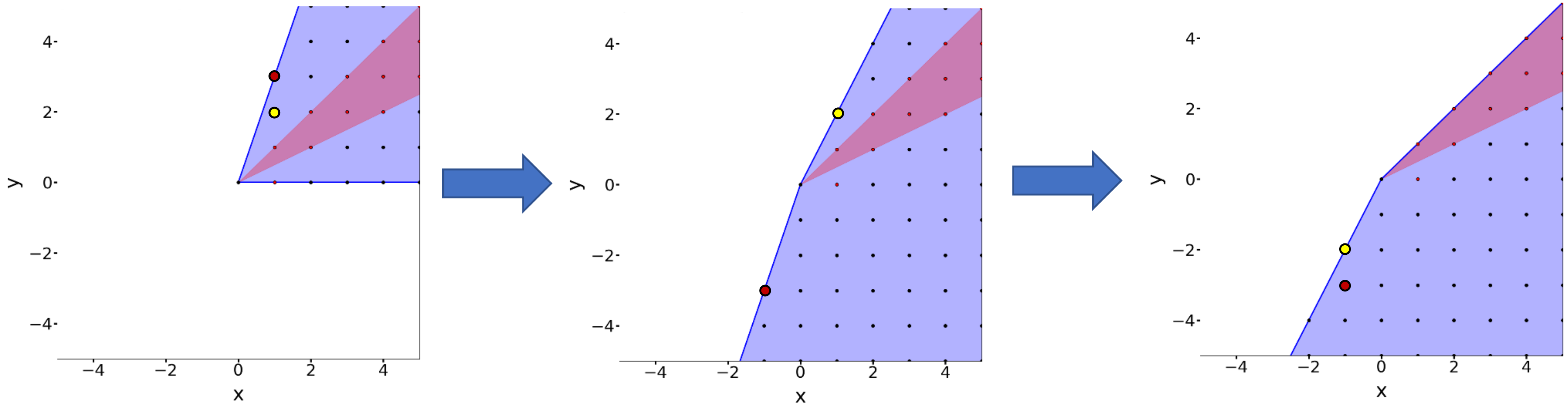

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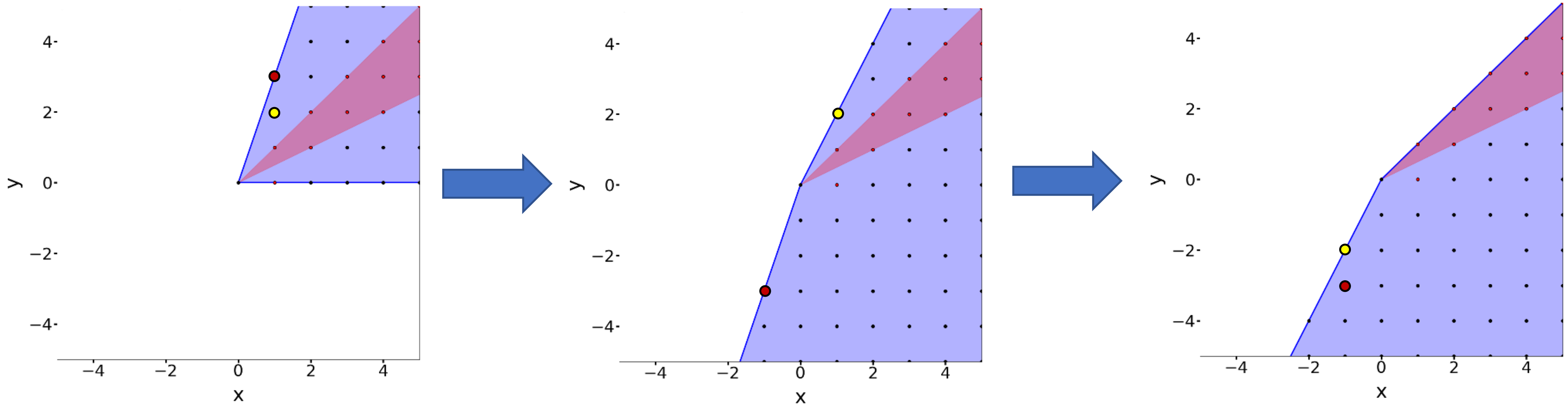

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Then we use the previous trick to compute GV invariants along those faces!

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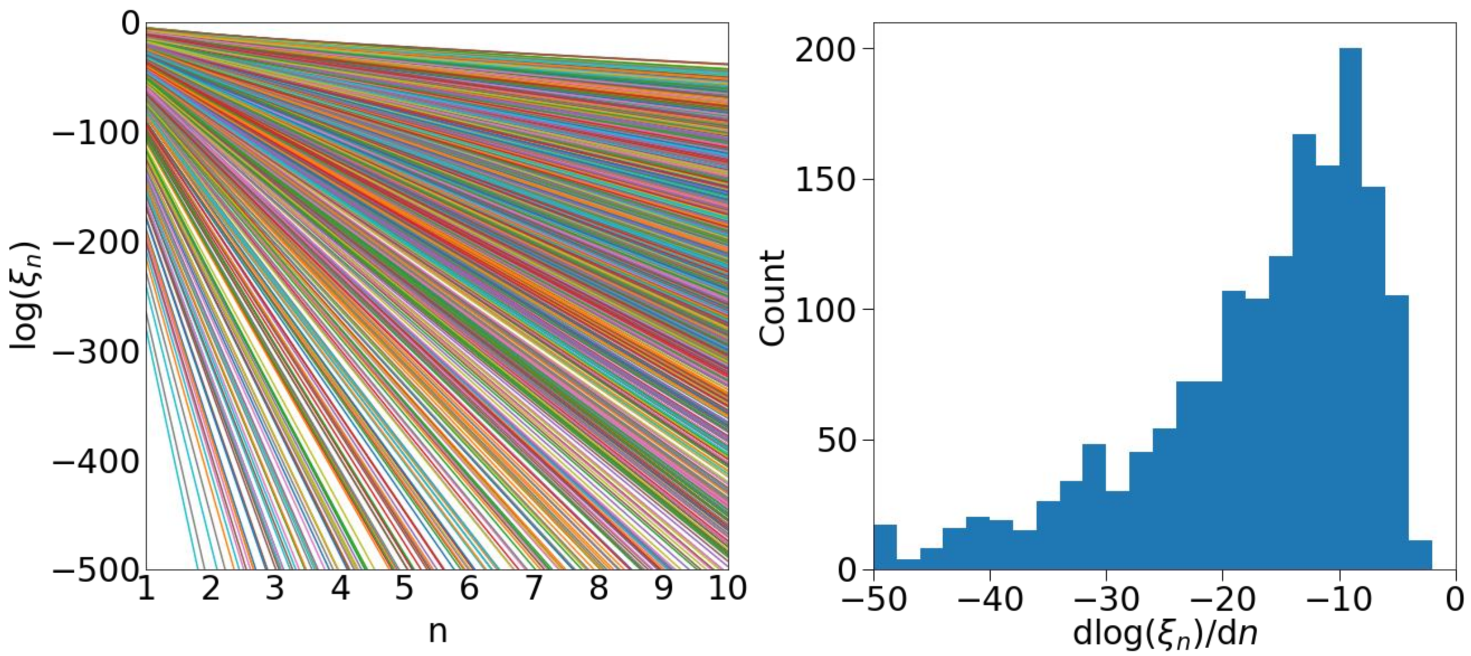
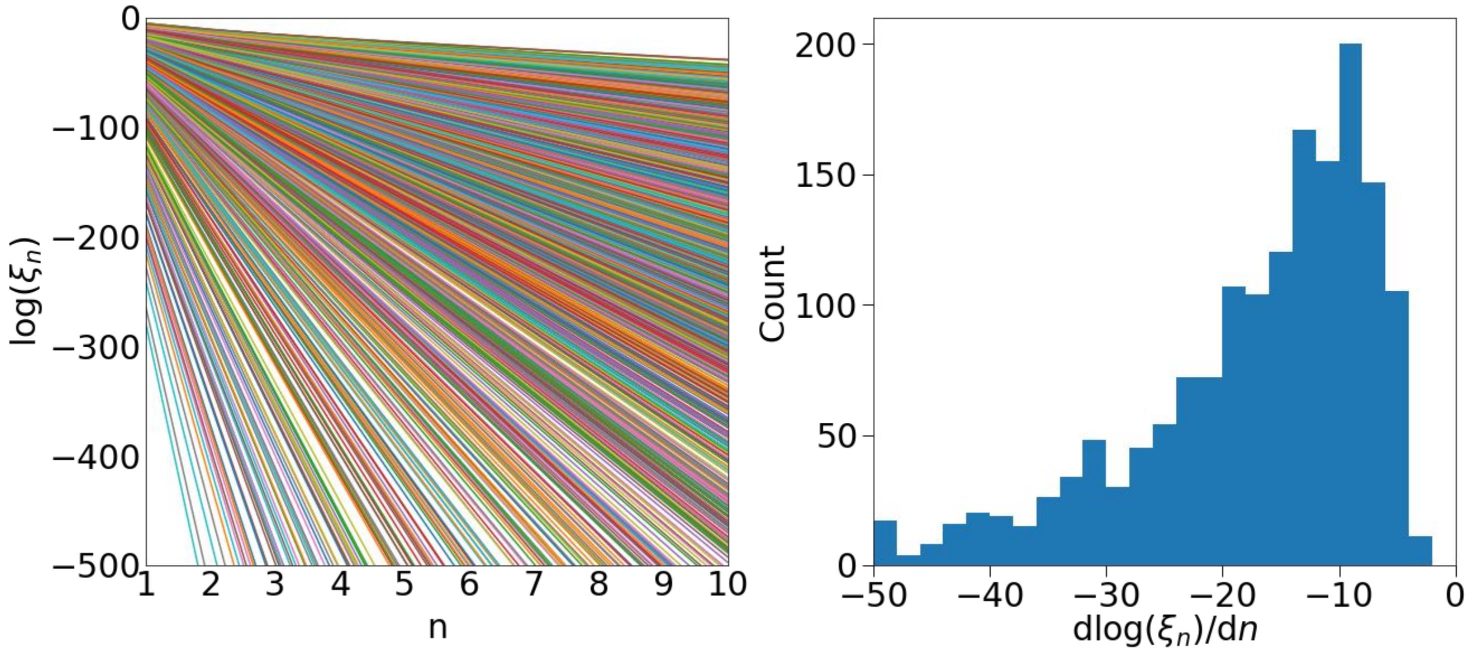


Do the sums converge?

Yes! In our flagship example we used a sample of 1728 potent rays along which we computed the GV invariants of the first 10 curve classes.

Here we plot the scaling of the contributions along the potent rays. $\xi_n = \mathrm{GV}_{n\vec{q}} e^{-2\pi n \vec{q}\cdot\vec{t}}$



The contributions decay exponentially, so the sum converges!

When can you get your hands on our computational tools?

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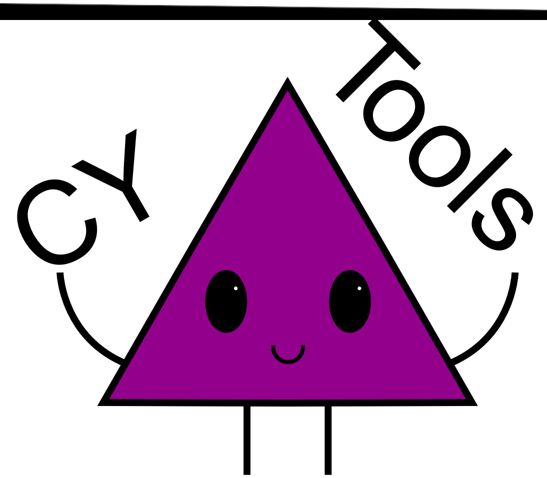

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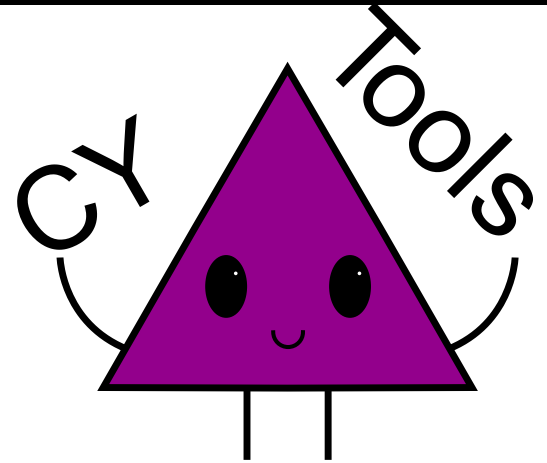

2019
• Any CY_3 from the KS database

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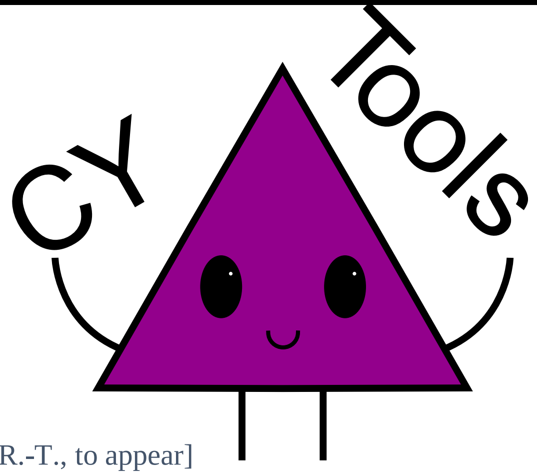

[Demirtas, McAllister, A.R.-T., to appear]

2019	Now
• Any CY_3 from the KS database	• Singular CYs • Nef-partitions, Toric CICYs • Fourfolds (and n-folds) • Many improvements

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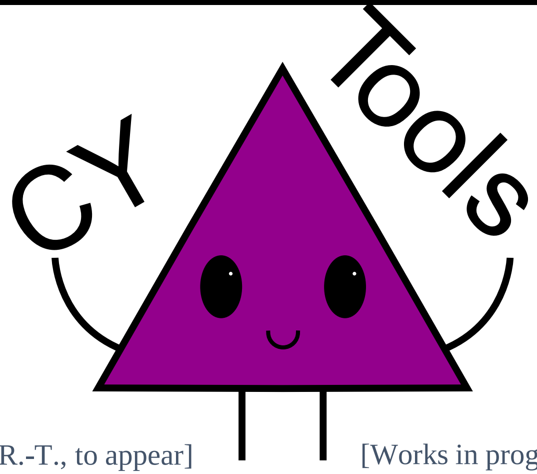
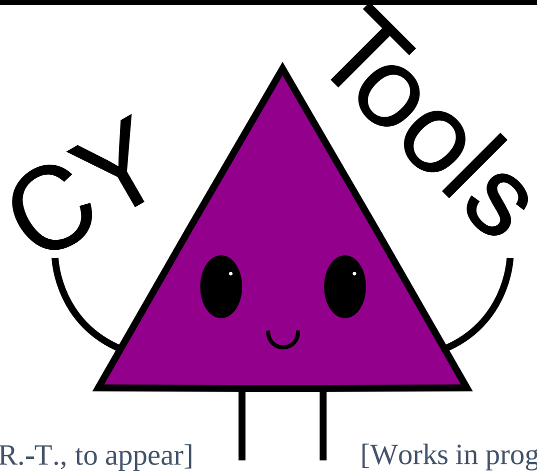


[Demirtas, McAllister, A.R.-T., to appear]

[Works in progress with Gendler, Kim, Kulkarni, Moritz, Nally, Stillman]

2019	Now	Soon
• Any CY_3 from the KS database	• Singular CYs • Nef-partitions, Toric CICYs • Fourfolds (and n-folds) • Many improvements	• GW/GV invariants • Orientifolding • Modular CYs • Non-favorable polytopes • Vacua finding

When can you get your hands on our computational tools?



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2019	Now	Soon	Future directions
• Any CY_3 from the KS database	• Singular CYs • Nef-partitions, Toric CICYs • Fourfolds (and n-folds) • Many improvements	• GW/GV invariants • Orientifolding • Modular CYs • Non-favorable polytopes • Vacua finding	• F-Theory • Heterotic string • VEX polytopes

When can you get your hands on our computational tools?

CY Tools

If any of this sounds interesting to you then let's talk!

[Demirtas, McAllister, A.R.-T., to appear]

[Works in progress with Gendler, Kim, Kulkarni, Moritz, Nally, Stillman]

2019	Now	Soon	Future directions
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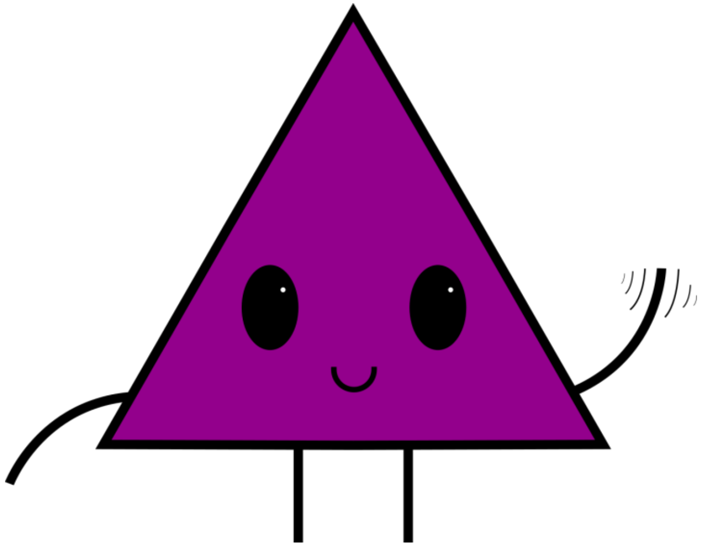
- We devised new approaches to test for the convergence of worldsheet instanton corrections at large number of moduli.
- The SUSY AdS vacua we constructed are in the radius of convergence of the instanton expansion.
- Our tools will be included in our CYTools package soon.

Thank you!



Questions?